COMPARATIVE STUDIES REVEAL PRINCIPLES OF MOVEMENT ON AND WITHIN GRANULAR MEDIA

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Abstract. Terrestrial locomotion can take place on complex substrates such as leaf litter, debris, and soil that flow or solidify in response to stress. While principles of movement in air and water are revealed through study of the hydrodynamic equations of fluid motion, discovery of principles of movement in complex terrestrial environments is less advanced in part because describing the physics of limb and body interaction with such environments remains challenging. We report progress our group has made in discovering principles of movement of organisms and models of organisms (robots) on and within granular materials (GM) like sand. We review current understanding of localized intrusion in GM relevant to foot and body interactions. We discuss the limb-ground interactions of a desert lizard, a hatchling sea turtle, and various robots and reveal that control of granular solidification can generate effective movement. We describe the sensitivity of movement on GM to gait parameters and discuss how changes in material state can strongly affect locomotor performance. We examine subsurface movement, common in desert animals like the sandfish lizard. High speed x-ray imaging resolves subsurface kinematics, while electromyography (EMG) allows muscle activation patterns to be studied. Our resistive force theory, numerical, and robotic models of sand-swimming reveal that subsurface swimming occurs in a "frictional fluid" whose properties differ from Newtonian fluids.

Key words. Locomotion, walking, running, crawling, swimming, lizard, robot, turtle, granular.

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1. Introduction.

Discovery of biomechanical [1, 2] and neuromechanical [3, 4] principles of locomotion in a given environment requires comparative study of organisms in the environment, understanding the mechanics of interaction with the environment, and modeling of both organism and environment. For example, an understanding of organism flight has emerged through identification of common patterns of movement [5] and experimental, computational and analytic analysis of air flow patterns which translate kinematics into dynamics. The extensive study of aerial and aquatic locomotion has resulted in major progress in modeling the interaction of organisms with and the creation of devices that can maneuver in fluid environments.

Principles that govern the locomotion of animals that live in complex *terrestrial* environments are much less understood. For example, mountain goats bound over steep rubble-strewn slopes with agility human-made devices cannot currently match [6]. One reason for this gap in understanding

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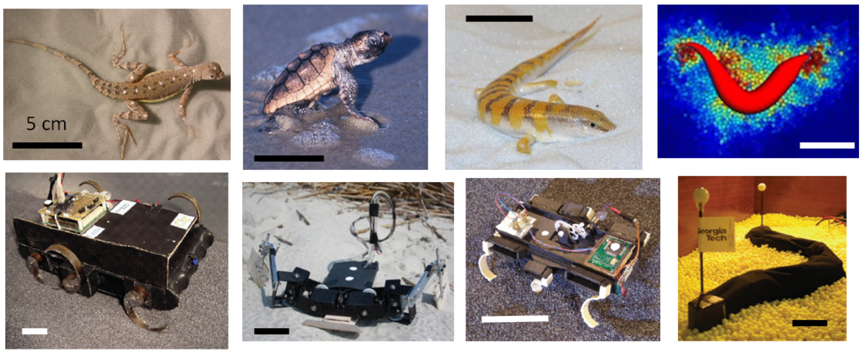

Fig. 1. *Organisms and models studied by our group to reveal principles of movement on and within GM. Top row: Zebra-tailed lizard, Loggerhead sea turtle (hatchling), Sandfish lizard, Sandfish numerical DEM simulation. Bottom row: SandBot, FlipperBot, RoACH, Sandfish robot. All scale bars are approximately* 5 *cm long.*

is that unlike aerial and aquatic environments, common terrestrial environments like dirt, leaf litter, rubble, and sand are not yet adequately described by models at a level comparable to those that describe fluid-flow (e.g., the Navier-Stokes equations). Prediction of ground reaction force is therefore a challenge, and, consequently, quantitative discovery of locomotor principles and construction of devices (like robots) that operate effectively in such environments remains elusive.

Dry granular media (GM), e.g., the sands of deserts and beaches, are common examples of complex flowing substrates. Composed of collections of particles that interact through dissipative contact forces, these materials exhibit both solid- and fluid-like features. GM are good substrates with which to study terrestrial locomotion since they are readily controlled (by use of a fluidized bed [7], for example) and thus repeatable initial conditions can be generated in the laboratory. A diversity of animals are important members of the ecosystems in sandy environments (examples in Fig. 1), and one can expect granular rheology to affect their locomotor strategies and performance. There have been many descriptive studies of organism behavior (e.g., [8, 9]) but fewer detailed studies [10, 11, 12] of above ground biomechanics on GM. While many organisms move over the surface of sand, a large number also bury in it (e.g., many lizards, snakes, scorpions, spiders, crabs) and some even swim *within* the sand [9]. However, because of limited visualization tools, there have been even fewer detailed subsurface studies [13].

In this overview, we discuss progress our group has made towards discovering principles of locomotion in dry GM by comparative studies of animals and models (physical and numerical) on and within GM substrates. We illustrate how our modeling approaches have advanced descriptions of limb and body interaction with GM. We first review the relevant physics of localized intrusion in GM and then discuss locomotion above and below the surface.

2. Localized intrusion of granular material. GM exhibit complex rheology [14] affected by both the properties of the particles (e.g., coefficient of friction, polydispersity, particle shape, etc.) and the compaction state of the medium. Compaction affects granular forces in non-trivial ways, but on average, closely packed GM resists larger stresses before flowing [15, 16]. Compaction can be characterized by the volume fraction, ϕ, the ratio of solid volume to occupied volume within a region. In terms of mass (m), occupied volume (V), and the particle density (ρ), $\phi = \frac{m}{\rho V}$. GM in nature such as on sand dunes [17] and beach environments [18] exist in a wide range of packing states; using a fluidized bed and dry, round particles, GM can be prepared [16] with packing states ranging from loose ($\phi \approx 0.58$) to close ($\phi \approx 0.64$) and corresponding mechanical properties similar to natural sand.

The frictional nature of GM produces a yield force (F_{yield}), a threshold below which grains do not flow in response to forcing [19]. Above F_{yield} GM flow and, for low intrusion speeds, the force on the intruder is speed independent [20] unlike the case for fluids. As intrusion speed increases inertial forces dominate frictional forces and typically vary as $F(v) \approx v^2$ [20]. Like the hydrostatic force in fluids, the average stress within GM increases approximately linearly with depth. Because the pressure drop across intruders is typically much smaller than the yield force, granular buoyancy in static granular beds is usually unimportant. Vertical intrusion of objects into GM results in a penetration force linear in depth z and projected intruder surface area A, namely $F(z) \approx \alpha A z$. The resistance of GM to penetration α is a function of the material properties and packing state.

The physics of GM most relevant to our studies is that of localized forcing: the penetration and movement of feet, limbs, heads, or bodies [7, 21, 18]. Studies of localized forcing with horizontally and vertically translating intruders in initially homogeneous GM have been conducted [22, 23, 24, 15]. Much like the case of fluids, intruders moving horizontally through GM experience drag and lift forces. In GM however, these forces arise from normal and frictional forces on the intruder's surface, which are supported by force chains between particles in the bulk [22]. Previous study showed that the drag force on an intruder horizontally translated through GM depend less on intruder shape as compared to that in fluid [23]. Simulation revealed that in GM a band-shaped region of grains flow upward and forward in front of the intruder, with a volume roughly proportional to intruder cross-sectional area regardless of intruder shape; the drag force whose magnitude is set by the weight of the grains within the flowing region is thus insensitive to intruder shape [25]. For arbitrary intruder shape, both drag and lift forces can be approximated by decomposing the leading surface into flat plates and summing the normal and tangential (frictional) forces on the plates. Since part of the grains can also be pushed upward or downward by the leading surface, the intruder may experience a net positive or negative lift force depending on its shape [25].

3. Limbed locomotion on sand: walking and running. To run, walk, or crawl on a substrate, terrestrial animals must generate forward thrust to advance and vertical thrust to counter gravity. Unlike on rigid, non-slip ground, on yielding substrates like sand animals must intrude their feet into the substrate to generate sufficient forces to generate forward (and vertical) propulsion. This creates a dilemma: deeper intrusion generates larger forces but the increased foot penetration reduces stride length and increases drag on the limbs and the body. Deeper penetration also results in increased energy loss due to irreversible work done on the substrate.

Our study of a six legged robot, SandBot (Fig. 1), demonstrates the precariousness of moving on GM [7] and the importance of GM's finite yield stress. In our study SandBot (~ 30 cm, ~ 2.3 kg) used an alternating tripod gait and relatively small, c-shaped legs (total area ≈ 15 cm^2). Since foot penetration forces are small at shallow depths, only when a large portion ($> 70\%$) of each leg intrudes into the substrate is sufficient vertical force produced to raise the body off the ground and move it forward. This is similar to walking on hard ground (i.e. inverted pendulum model [1]), except that the foothold is deep relative to leg length and between steps the body is supported by the ground surface. SandBot can walk at speeds up to 30 cm/s (~ 1 bl/s) on sand or "swim" slowly (~ 1 cm/s). Only close to a particular set of limb kinematics (defined by stance duration, stance location, and duty factor) does it walk with little slip. Optimal limb kinematics are determined by forces generated during *rotational intrusion* into GM [26], and these forces differ from those generated during either pure vertical penetration [27] or horizontal drag [16]. During rotational intrusion, the maximum force occurs $\approx 40^\circ$ before the point at which the limb is deepest in the ground.

With the optimal limb kinematics, SandBot's speed increases sublinearly with stride frequency and volume fraction ϕ, but for sufficiently low ϕ and/or high stride frequency, the speed is small (~ 1 cm/s). A model [7] reveals that the mechanism of effective movement on GM relies on solidification and explains the sublinear increase: since penetration force increases as $\alpha A z$, the limb penetrates into the material to a depth governed by the body weight and the inertial forces needed to accelerate the body to limb velocity. At the depth where this force balance is achieved, the material under the limb solidifies and the robot "rotary" walks, with the limbs rotating atop the solidified grains and moving the body forward by a distance determined by leg geometry. As ϕ decreases and/or stride frequency increases, penetration depth increases to the point where step length (inversely related to penetration depth) is smaller than leg length. Consequently, over consecutive steps the legs encounter previously disturbed ground which does not provide as much penetration resistance. The robot can no longer raise its body over a solid foothold, and instead "swims" forward slowly ($\sim$ 1 cm/s) via drag on its legs moving through grains that are always locally fluidized.

Our study of SandBot provides broader insights into the principles governing effective locomotion on sand. Since surface penetration is required for locomotion on GM yet reduces stride length and causes increased drag, it is advantageous for an animal (or a legged robot) to have large feet and long legs and use appropriate kinematics to reduce relative leg penetration while generating the required thrust. The zebra-tailed lizard (*Callisaurus draconoides*, $\sim$ 10 cm, $\sim$ 10 g, Fig. 1), a desert generalist, provides an excellent realization of these design criteria [28]. It can run at speeds up to $\sim$ 4 m/s ($\sim$ 50 bl/s) on hard ground [29] and $\sim$ 2 m/s (25 bl/s) on sand [12]. Compared to closely-related lizards of similar size, it has the longest hind limbs ($\approx$ 90% body length) and the largest hind feet (total area ≈ 2 cm^2) with extremely elongated toes [29]. While standing on loose sand, the foot only penetrates the surface by $\approx$ 3 mm or 10% of vertical leg length ($\sim$ 3 cm). High speed x-ray imaging reveals that during running on sand, the foot impacts the surface with a plantigrade foot posture to create the largest projected foot area and penetrates a maximal depth of $\approx$ 1.3 cm in stance; a majority ($>$ 50%) of the leg stays above surface, making it possible for the animal to take long strides with reduced drag. Recently developed small robots like DASH and RoACH (Fig. 1, $\sim$ 10 cm, $\sim$ 20 g) are approaching the organisms in locomotor performance – like the zebra-tailed lizard, they have relatively large feet (total area ≈ 3 cm^2) for their body weight, and can run at $\sim$ 10 bl/s on sand with appropriate foot design [30].

4. Limbed locomotion on sand: crawling. Certain aquatically adapted organisms (e.g., mudskippers, sea lions, sea turtles) use paddle-like appendages (flippers) to crawl on deformable terrestrial materials [18, 31]. We have studied the terrestrial locomotion of sea turtles, animals constrained by an evolutionary life history to come ashore and lay nests near sand dunes [32]. While crawling on sandy beaches using aquatically adapted limbs, a sea turtle rests most of its body weight on its flat plastron, inserts its paddle-like flippers into sand, and pushes horizontally (and vertically) to generate enough thrust to overcome belly friction and accelerate its body. Both adults ($\sim$ 100 kg) and hatchlings ($\sim$ 20 g, Fig. 1) employ this strategy.

Since field studies can elicit behaviors not demonstrated in the laboratory [33], we conducted a study of locomotion performance and limb-ground interaction of hatchling Loggerhead Sea turtles (*Caretta caretta*) in the field on Jekyll Island, GA, USA [18]. A field portable fluidized bed trackway allowed us to mimic the beach substrate in a controlled fashion by varying the compaction state of the sand and the incline angle [7, 18]. High speed imaging under infrared lights revealed that during rapid runs (3 bl/s over the deformable sand–comparable to the speed on hard ground) the limbs exhibited minimal to no slip [18]; the animals did not paddle through sand. During each step the forward speed increased to a maximum followed by a decline to zero [18]. On sand, the flipper penetrated the material and the wrist bent, solidifying the material behind the flipper during the thrust phase. This solidification process during forward movement results in a no-slip condition such that thrust forces stay below the material yield force ($F_{thrust} < F_{yield}$) and allows sea turtles to move effectively on sand.

To begin to systematically explore crawling on sand using paddle-like limbs, we developed a physical model of the sea turtle (FlipperBot, Fig. 1). Its performance is sensitive to belly friction, suggesting the importance of lift and forward thrust generation during a step. FlipperBot has a wrist that can be made flexible or rigid. Our initial results show that a flexible wrist allows solidification of material during forward motion, and improves locomotor performance. We attribute the performance increase to a decrease in the amount of material disturbed during a step by the flexible limb: this reduces the probability of encountering previously disturbed ground during the next step which, if it occurs, reduces forward progress.

5. Undulatory swimming in a frictional fluid: kinematics. We now discuss the principles that allow organisms to "swim" within GM [8]. We used high speed x-ray imaging to study a small ($\sim 8\,\mathrm{cm}$) desert-dwelling lizard, the sandfish (Fig. 1), which inhabits the Saharan desert of Africa and moves within GM of different ϕ. Once subsurface the animal no longer used limbs for propulsion. Instead it placed its limbs against its sides and executed an undulatory motion of the body with large amplitude axial oscillation, using the body to propel itself at speeds up to $\sim 2\,\mathrm{bl/s}$. The sandfish dove into the material at an angle of $22 \pm 3.7^\circ$ to a depth of $2.1 \pm 0.5\,\mathrm{cm}$.

Subsurface swimming kinematics were well characterized by a traveling wave sinusoidal wave propagating along the body from head to tail: $y = A \sin \frac{2\pi}{\lambda}(x + v_w t)$, with y the displacement from the mid-line of a straight animal, A the amplitude, λ the wavelength, f the wave frequency, $v_w = f\lambda$ the wave speed, t the time, and x the distance along a line joining the end points of the animal and parallel to the direction of motion. In both low and high ϕ GM (in both 0.3 mm and 3 mm glass beads) the ratio of A to λ was approximately 0.2. For each condition tested the animal increased its forward speed by increasing its undulation frequency [21, 34].

The forward speed of the sandfish, v_x, was not equal to the speed of the wave traveling along its body, but instead was equal to the product of f and λ times a constant factor η, so that $v_x = \eta f \lambda$ with $\eta = 0.53 \pm 0.04$ (in 0.3 mm glass particles). Tracer particles placed in the GM revealed a backward flow of grains as the animal moved forward. Slipping while progressing is common to undulatory swimmers in deformable media over a wide range of length scales (e.g., eels and spermatozoa in fluids [1]) and is characterized by the wave efficiency, $\eta = v_x/v_w$ [35], defined as the ratio between the forward speed of the animal, v_x, and the velocity of the wave traveling down its body, v_w. Remarkably, η in 0.3 mm and 3 mm particles was approximately 0.5, and was independent of volume fraction ϕ despite differences in drag resistance of nearly a factor of two between the highest and lowest ϕ states.

6. Undulatory swimming: muscle activity. To develop neuromechanical models [36, 4] of locomotion we must translate kinematics into forces (dynamics) and ultimately connect to the neuromuscular control system (including actuation and sensing). To learn how the sandfish generates force during sand-swimming, we studied the activation pattern in its epaxial musculature using electromyogram (EMG) recordings synchronized with high speed x-ray and visible light imaging [37]. We hypothesized that in such a frictional, highly dissipative environment the activation strategy would be dictated by the speed independent but depth dependent forces in GM.

To swim subsurface in GM, the sandfish generates an anterior-to-posterior wave of muscle activation. EMG onset occurs at or just prior to the maximal convexity of the body (the point at which the muscle is maximally stretched), with more posterior muscles activated earlier in the muscle strain cycle. This timing pattern is in accord with the activation timing found for undulatory swimmers in water [38]. As a sandfish dives farther into the GM, EMG intensity monotonically increases with undulation number and, therefore, depth. For a given undulation number, EMG intensity is independent of speed; to move faster the sandfish simply propagates the wave faster. These results support the hypothesis that sandfish require a higher amount of muscle force with increasing depth but not speed. Despite an increase in resistance force measured in drag experiments at higher ϕ, at shallow depths EMG intensity did not depend on ϕ [37].

7. Modeling sand-swimming: simulation, RFT, and robots. Challenged by the lack of constitutive equations for GM, we have modeled sand-swimming in three ways: numerically, physically (robot), and analytically using a Resistive Force Theory (RFT). All modeling methods demonstrate that movement can be thought of as occurring within a “frictional” fluid where force is dominated by frictional contacts within the material locally flowing around the body. The models demonstrate the differences and similarities of swimming in GM compared to swimming in fluids.

We use a 50-segment numerical simulation to model the sandfish with relative angles between segments specified to generate a traveling wave as observed in the animal [34]; the center of mass position and inter-segment torques are unconstrained. The GM is modeled using an experimentally validated Discrete Element Model (DEM) [39] of $\sim 10^5$, 3 mm diameter particles (Fig. 1). Particle-particle and particle-intruder interactions include repulsive and viscous forces in the normal direction, and a frictional force in the tangential direction. The numerical model quantitatively reproduces kinematic features of the locomotion (e.g., speed vs. frequency). Examination of the flow of particles around the animal body supports the frictional fluid picture.

The DEM model also reproduces the observed muscle activation patterns: when constrained to dive at an angle, activation torque in the model increases with depth but is independent of speed. Force on the body is independent of undulation frequency which results in a frequency independent cost of transportation. The DEM model reveals only a small difference in motor torque in closely and loosely packed media exists at any depth. The similarities between simulation and experiment imply that the observed biological muscle activation pattern results from the interaction with GM.

Since the organism swims within a localized fluid, we use a RFT model [40, 21] originally developed for low Reynolds number swimmers to gain insight into swimming in the granular medium. In the RFT, the body of the organism is partitioned into infinitesimal elements along its length. When moving relative to the medium, each element experiences resistive thrust and drag forces. Resolving these forces into perpendicular and parallel components and balancing them by integrating forces over the length of the body (and head) predicts forward swimming speed at a given frequency. Since at biologically relevant swimming speeds ($0 - 0.4$ m/s) force is independent of speed [21, 20], the force on an element can be characterized as a function of only the *direction* of the velocity relative to its orientation. As the entry angle of the animal is small ($< 30°$), we approximate the motion of the animal as occurring in the horizontal plane.

Since resistive force laws in GM are not available, we measured the forces on rods with comparable cross sections to the animal body as the rods were dragged through GM at a fixed depth. With these force laws, the RFT agrees well with the DEM model. The angular dependence of the force laws in GM resembles the forces generated in a Newtonian fluid at low Reynolds number: the perpendicular force increases and the parallel decreases with the angle between the velocity of the rod and its longitudinal axis. However, while the functional forms of the forces in low Re can be approximated as sines and cosines, in GM, they do not have these simple functional forms. Further the ratio of the average magnitude of the perpendicular forces to the parallel forces is larger in GM ($> 3 : 1$) than in fluid ($\approx 2 : 1$). Consequently, thrust is relatively larger in GM compared to that in a fluid at low Reynolds number. The difference in force laws explains the higher η observed for sandfish (≈ 0.5) compared to non-inertial low Re swimmers in fluids (≈ 0.2). The RFT also suggests that the packing state does not affect η (or net torque) because both thrust and drag scale similarly with changes in packing. An alternative explanation is that the material disturbed by the sandfish rapidly evolves to the same ϕ_c, the critical volume fraction [15, 16], and thus the body of the organism always moves within GM of the same ϕ.

Finally we developed and tested a physical model, the sandfish robot (Fig. 1). The robot consists of seven coupled motors, each with a single joint that permits angular excursions in the body plane, connected to each other by identical links to form the body [34]. We commanded the observed sandfish kinematics using an open-loop controller. Like the animal, the forward velocity of the robot monotonically increased with increasing frequency. However, for the robot $\eta = 0.34 \pm 0.03$, significantly below the values measured for the animal in experiment and predicted by the RFT and DEM simulation (with 50 segments). We found by simulating the robot that this smaller η was due to the robot's relatively few segments. Increasing the number of segments while keeping body length fixed caused η to increase, plateauing at a value equal to the animal ($\eta \approx 0.5$) above 15 segments.

All three models predict that η increases with increasing A, but we observed that the animal does not operate at high A. The RFT model shows that operating at large A comes at a cost: since the animal's length is fixed, its wavelength λ decreases with increasing A. Because $v_x = \eta\lambda f$, as A increases, the competition between increasing η and decreasing λ results in a maximum in forward displacement per cycle at $A/\lambda \approx 0.2$. This finding is captured by all the models. The biological data reside close to the peak of the curve, indicating that the animal is maximizing its sand-swimming speed in accord with the hypothesis that the sandfish's rapid burial and swimming behavior is an escape response [8].

8. Conclusions and outlook. We have briefly described our efforts to discover principles of movement on and within GM. Discovery of principles of movement requires appropriate models of substrate interaction, which in GM include phase transitions, speed independent but depth dependent forces, and dependence on initial conditions (e.g., ϕ). Above ground, movement of some legged locomotors can be enhanced by maintaining solidification which suggests biological hypotheses for control of limb movement. Our walking and crawling models provide insights into the body and limb design and strategies for effective movement on the surface of sand. Subsurface swimming of the sandfish lizard and related models reveal that movement occurs within a "frictional" fluid whose properties dominate the locomotor pattern and activation strategy. Our RFT, numerical, and physical models predict kinematics and motor patterns for rapid sand-swimming. Our results are a step in developing interaction rules and principles of movement of organisms in even more complex terrestrial environments (flowing/solidifying materials composed of more complex elements, like rubble and leaf litter) and will enable construction of devices that can maneuver robustly in such terrain.

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